Solutions for Homework Set 6

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1. *Kittel Problem 8.1*

Solution

(a) We are given $E_g = 0.23\text{eV}$, $\epsilon = 18$, and $m_e = 0.015m_e$. The donor ionisation energy is given by Kittel (8.51):

$$E_d = \frac{e^4 m_e}{2(4\pi\epsilon\varepsilon_0\hbar)^2} = \frac{13.6 m_e}{\epsilon^2 m}\text{eV}.$$

With the given values we have

$$E_d = 0.630\text{meV}. \tag{1}$$

As a check, Si has a ionisation energy of $E_d = 29.8\text{meV}$, much greater than the donor ionisation energy as expected.

(b) Similarly the Bohr radius is given by (8.52) in *Kittel*. This is

$$\begin{aligned} a_d &= \frac{0.52\epsilon}{m_e/m}\text{Å}, \\ &= 636\text{Å}. \end{aligned} \tag{2}$$

(c) Although it is impossible to calculate the exact answer without without knowing the crystal structure of indium antimonide we can give a reasonable guess assuming a uniform distribution of donor atoms. In this case there is overlap when there is more than one atom in a sphere of radius a_d. Hence, there is overlap when

$$N_D\left(4\pi a_d^3/3\right) \geq 1. \tag{3}$$

Hence,

$$N_D \geq 9.38 \times 10^{20}\text{m}^{-3}. \tag{4}$$

2. *Kittel Problem 8.2*

Solution

We are given some semiconductor with concentration $N_D = 10^{19}\text{m}^{-3}$ of donor states, an ionisation energy of $E_d = 1\text{meV}$ and effective mass of $m_e = 0.01m$.

(a) At $T = 4\text{K}$, we have $k_B T = 0.345\text{meV}$, and $k_b T \ll E_d$. Hence, using (8.53) in *Kittel*:

$$n \approx (n_0 N_d)^{1/2} \exp\left(-E_d/2k_b T\right),$$

gives with $n_0 = 3.85 \times 10^{19}$:

$$n \approx 4.61 \times 10^{18}\text{m}^{-3}. \tag{5}$$

(b) The Hall coefficient is given by *Kittel* (6.55):

$$R_H = \frac{E_y}{j_x B} = -\frac{1}{ne}, \tag{6}$$

where the second equality follows as $E_g \gg k_b T$, and we assume the free electron model studied in Chapter 6 of *Kittel*. Using the values above gives

$$R_h = -1.35\text{m}^3\text{C}^{-1}. \tag{7}$$

3. *Kittel Problem 8.3*

Solution

We let the static magnetic field lie in the z direction, and the current flow in the x direction. We assume equilibrium is established so that there is no current in the y direction. From the last HW problem set, problem 6.9 in *Kittel* gives

$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \sigma_e \begin{pmatrix} 1 & -\omega_c\tau_e & 0 \\ \omega_c\tau_e & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}, \tag{8}$$

where in the low B field limit we ignore $(\omega_c\tau)^2$ and B^2 terms.

To include the current due to the presence of the holes we include a similar conductivity term with e replaced by $-e$. This is justified because a hole acts like an electron with a positive electric charge (See *Kittel* (8.22)). Indeed one can follow through the derivation in problem 6.9 with $e \to -e$, $m_e \to m_h$, $\tau_e \to \tau_h$ giving

$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \sigma_h \begin{pmatrix} 1 & \omega_c^*\tau_h & 0 \\ -\omega_c^*\tau_h & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}, \tag{9}$$

where $\omega_c^* = eB/m_h$, $\sigma_h = ne^2\tau_h/m_h$. The total current is given by the sum of these two terms. As $E_z = 0$, we have $j_z = 0$, and so

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \sigma_e + \sigma_h & \sigma_h\omega_c^*\tau_h - \sigma_e\omega_c\tau_e \\ -\sigma_h\omega_c^*\tau_h + \sigma_e\omega_c\tau_e & \sigma_e + \sigma_h \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}. \tag{10}$$

In terms of the mobility $\mu_e = e\tau_e/m_e$, and $\mu_h = e\tau_h/m_h$ we have

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = e \begin{pmatrix} (n\mu_e + p\mu_h) & -B(n\mu_e^2 - p\mu_h^2) \\ B(n\mu_e^2 - p\mu_h^2) & (n\mu_e + p\mu_h) \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}. \tag{11}$$

The equilibrium condition that $j_y = 0$ implies that

$$E_x = -\frac{n\mu_e + p\mu_h}{B(n\mu_e^2 - p\mu_h^2)} E_y. \tag{12}$$

Then solving for j_x gives us

$$j_x = -\frac{e}{B}\frac{(n\mu_e + p\mu_h)^2}{(n\mu_e^2 - p\mu_h^2)} + \mathrm{O}(B). \tag{13}$$

Then the Hall coefficient is given by

$$R_H = \frac{E_y}{j_x B} = -\frac{n\mu_e^2 - p\mu_h^2}{e(n\mu_e + p\mu_h)^2}. \tag{14}$$

Introducing $b = \mu_e/\mu_h$ we then have

$$R_H = \frac{p - nb^2}{e(nb + p)^2}. \tag{15}$$

4. *Kittel Problem 8.4*

Solution

The equation of motion (8.6) in *Kittel* gives us

$$\hbar\frac{\mathrm{d}\vec{k}}{\mathrm{d}t} = -e\vec{v} \times \vec{B},$$

where $\vec{v} = \nabla_{\vec{k}}\epsilon(\vec{k})/\hbar$. The relevant energy surface is given by

$$\epsilon(\vec{k}) = \hbar^2\left(\frac{k_x^2 + k_y^2}{2m_t} + \frac{k_z^2}{2m_l}\right), \tag{16}$$

and so

$$\vec{v} = \hbar \left(\frac{k_x}{m_t} \hat{x} + \frac{k_y}{m_t} \hat{y} + \frac{k_x}{m_l} \hat{z} \right).$$

The system is symmetric about the z axis, and so without loss of generality we can orentiate the magnetic field in the x-direction. Hence, the equations of motion become

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} k_x \\ k_y \\ k_z \end{pmatrix} = -e\hbar \begin{pmatrix} 0 \\ k_z B_x / m_l \\ -k_y B / m_t \end{pmatrix}, \tag{17}$$

which are easily solved and are harmonic in nature. Then, solving for either k_y or k_z gives the same equation:

$$\frac{\mathrm{d}^2 k_y}{\mathrm{d}t^2} = -\left(\frac{e^2 B^2}{m_t m_l} \right) k_y,$$

which has solution $k_y = A \sin \omega_c t + B \cos \omega_c t$, where

$$\omega_c = \frac{eB}{(m_l m_t)^{1/2}}. \tag{18}$$